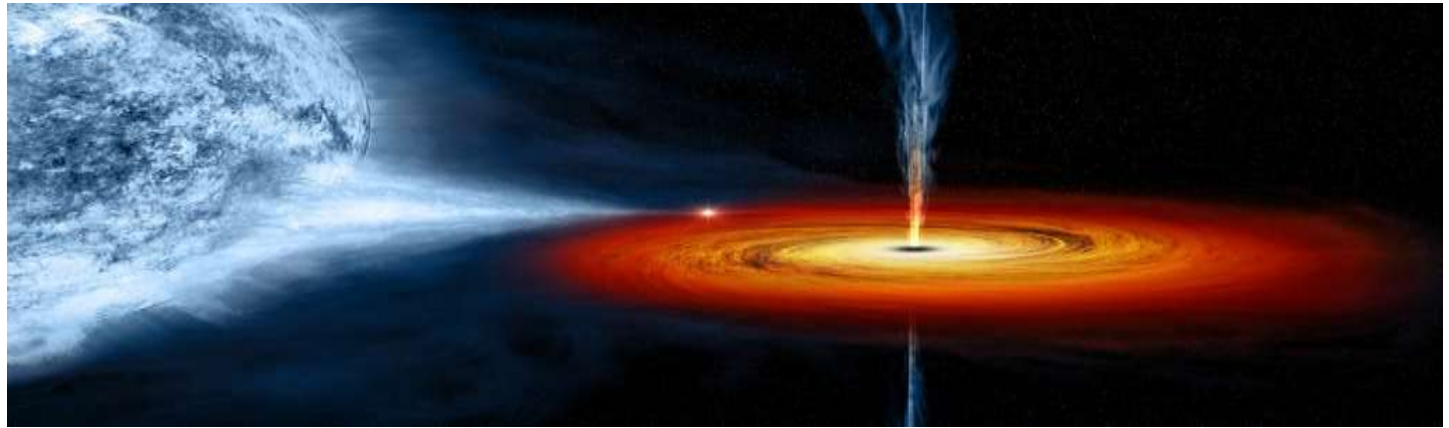


Restrictions on Spacetimes around Rotating Neutron Stars (Rossi X-ray Timing Explorer)

Gabriel Török

*Institute of Physics, Research Centre for Computational Physics and Data
Analysis, Silesian University in Opava, Czech Republic*



1. Fundamental theories vs. LMXBs spectra and timing

Artists view of LMXBs

“as seen from a hypothetical planet”

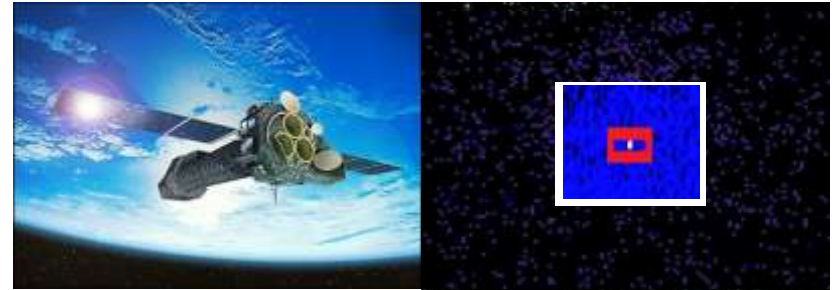


Compact object:

- Black hole or neutron star



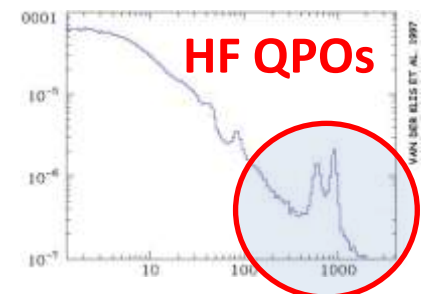
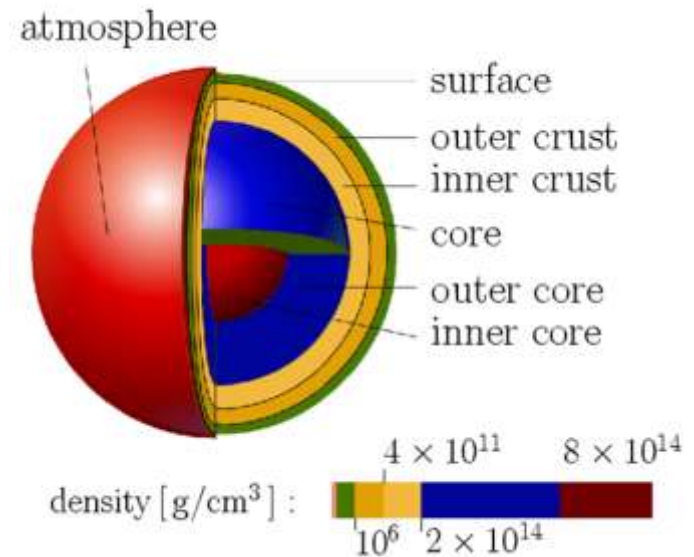
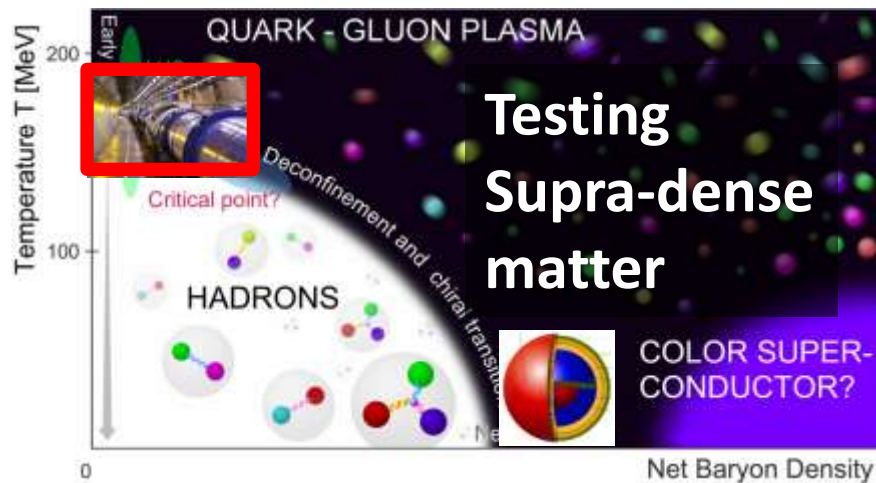
$T \sim 10^6 \text{K}$
>90% X-ray



Observation and theory: The X-ray radiation absorbed by Earth atmosphere is studied using detectors on orbiting satellites. The observations allows to test fundamental physical theories under extreme conditions that are impossible to reach in terrestrial laboratories.

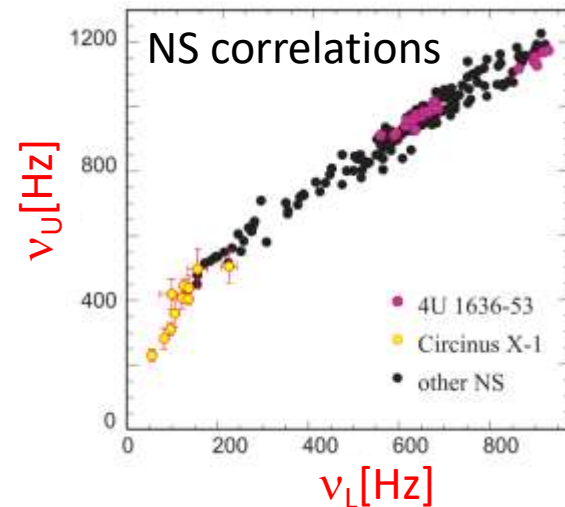
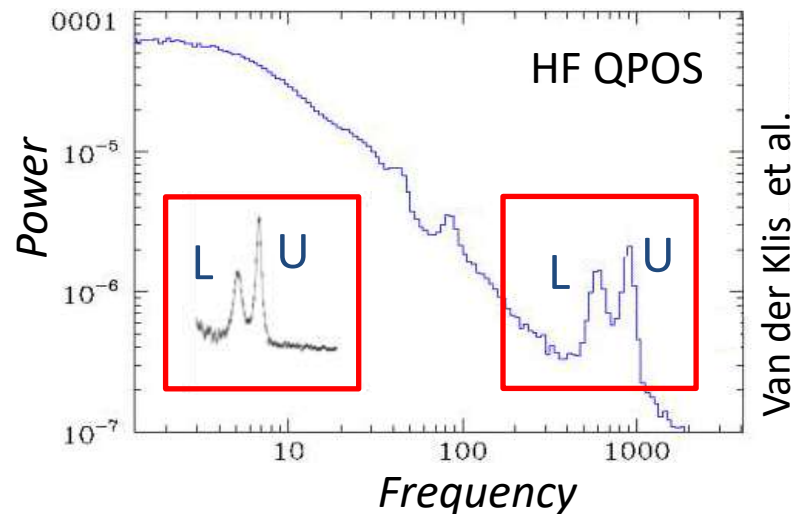
1. Motivation: fundamental theories vs. LMXBs spectra and timing

LMXBs provide a unique chance to probe effects in the strong-gravity-field region. The way how electromagnetic radiation propagates in space, its time variability and shape of lines in its energetic spectrum are invaluable probes to physical behavior of the matter in strong gravitational field. Similarly, a systematic study of the properties of neutron stars allows us to explore supradense matter.



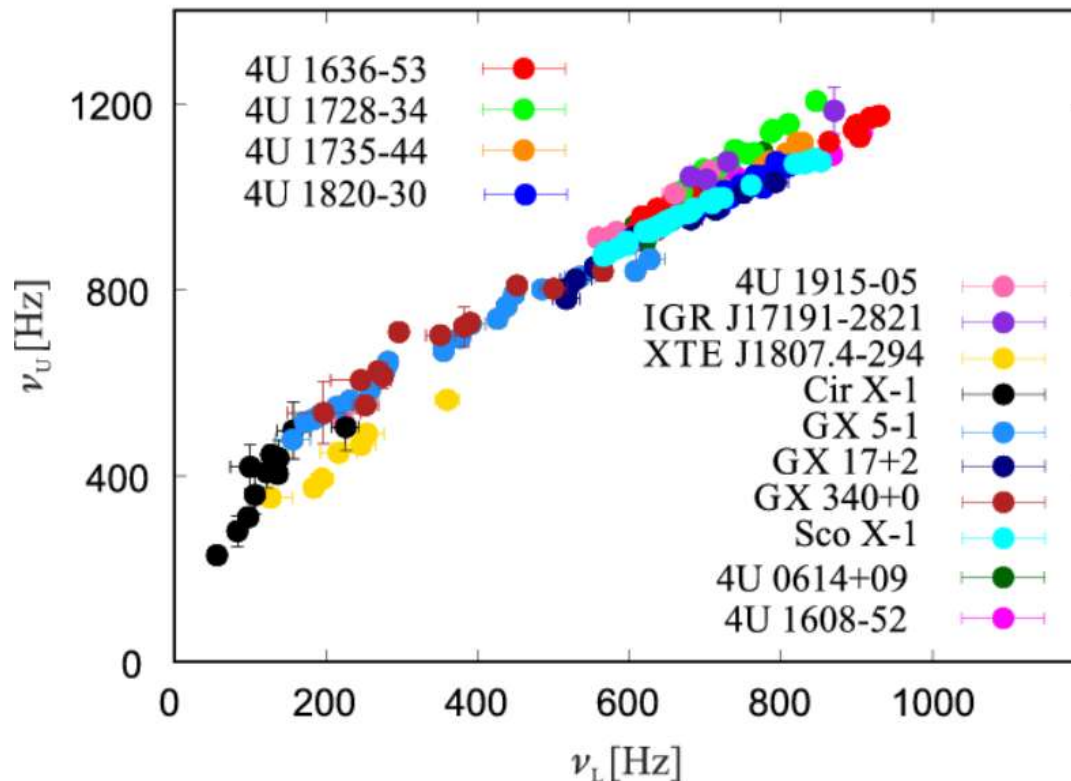
2. High frequency QPOs in NS LMXBs

HF QPOs appear in X-ray fluxes of several LMXB sources. Commonly to BH and NS they often behave in pairs. There is a large variety of ideas proposed to explain this phenomenon. Most often, QPOs are related to orbital motion in **strong gravity**. There is a belief that gravitational field and NS properties can be studied using QPO observations, e.g., using their frequency correlations.

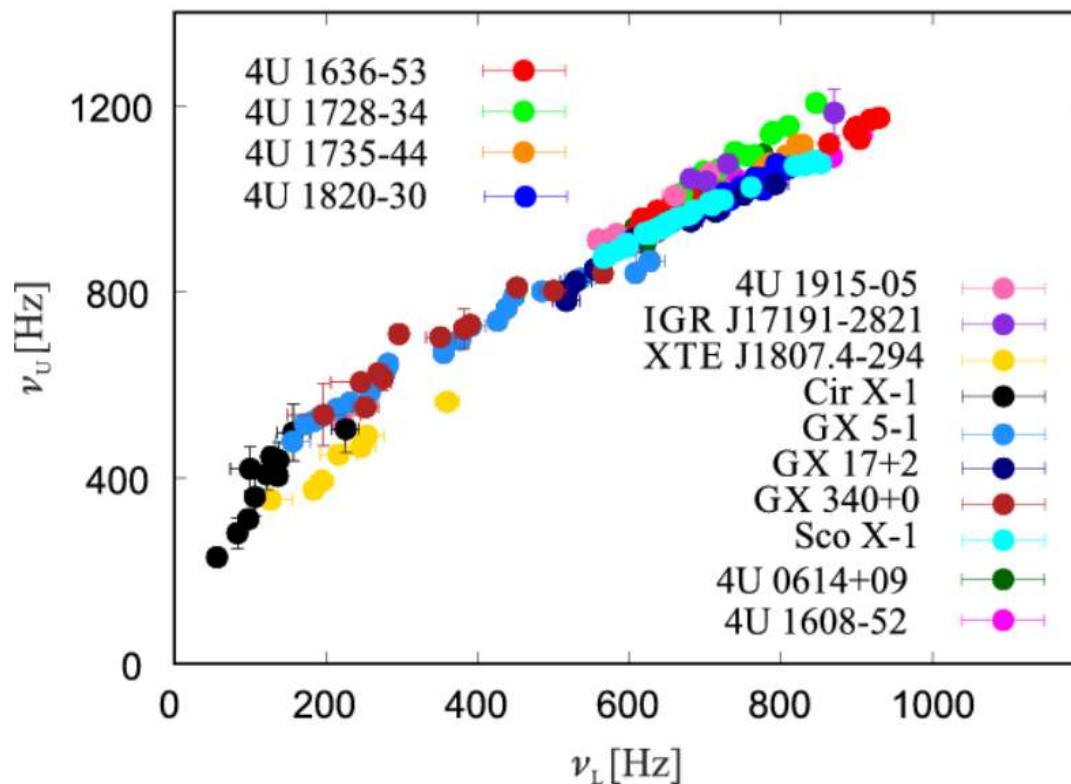


2. High frequency QPOs in NS LMXBs

RXTE operating from 1995 till 2012 has provided a high amount of NS data. Timing analysis of X-ray fluxes of more than dozen of NS systems reveals remarkable correlations between the frequencies of two characteristic peaks present in the power-density spectra.



2. High frequency QPOs in NS LMXBs



Several models have been proposed.

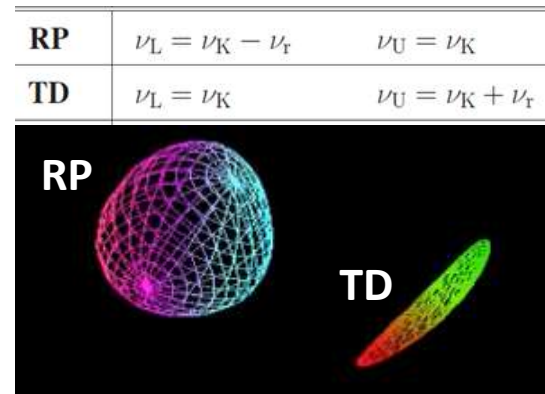
[e.g., Alpar & Shaham (1985); Lamb et al. (1985); Stella et al. (1999); Morsink & Stella (1999); Stella & Vietri (2002); Abramowicz & Kluzniak (2001); Kluzniak & Abramowicz (2001); Abramowicz et al. (2003a,b); Titarchuk & Kent (2002); Titarchuk (2002); Kato (1998, 2001, 2007, 2008, 2009a,b); Meheut & Tagger (2009); Miller et al. (1998a); Psaltis et al. (1999); Lamb & Coleman (2001, 2003); Kluzniak et al. (2004); Abramowicz et al. (2005a,b), Petri (2005a,b,c); Miller (2006); Stuchlík et al. (2007); Kluzniak (2008); Stuchlík et al. (2008); Mukhopadhyay (2009); Aschenbach 2004, Zhang (2005); Zhang et al. (2007a,b); Rezzolla et al. (2003); Rezzolla (2004); Schnittman & Rezzolla (2006); Blaes et al. (2007); Horak (2008); Horak et al. (2009); Cadez et al. (2008);]

3. Orbital models of HF QPOs and NS parameters

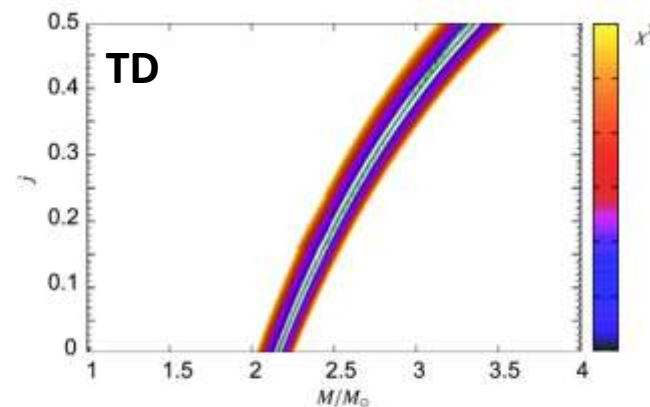
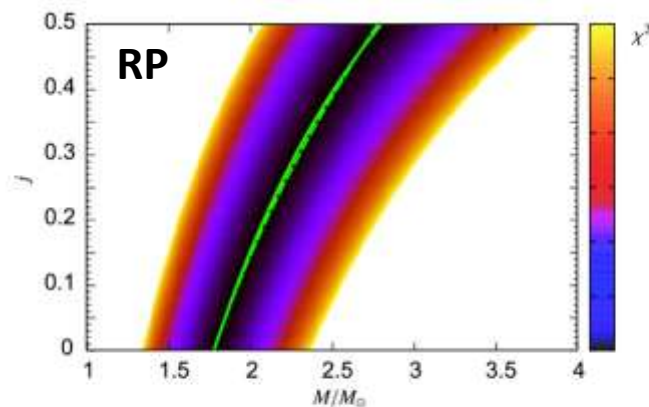
Models: Most of the models relate QPOs to the orbital motion. For instance, “blob” models (Stella et al., Cadez et al.,...).

It is possible to compare the model and data (here atoll source 4U 1636-53).

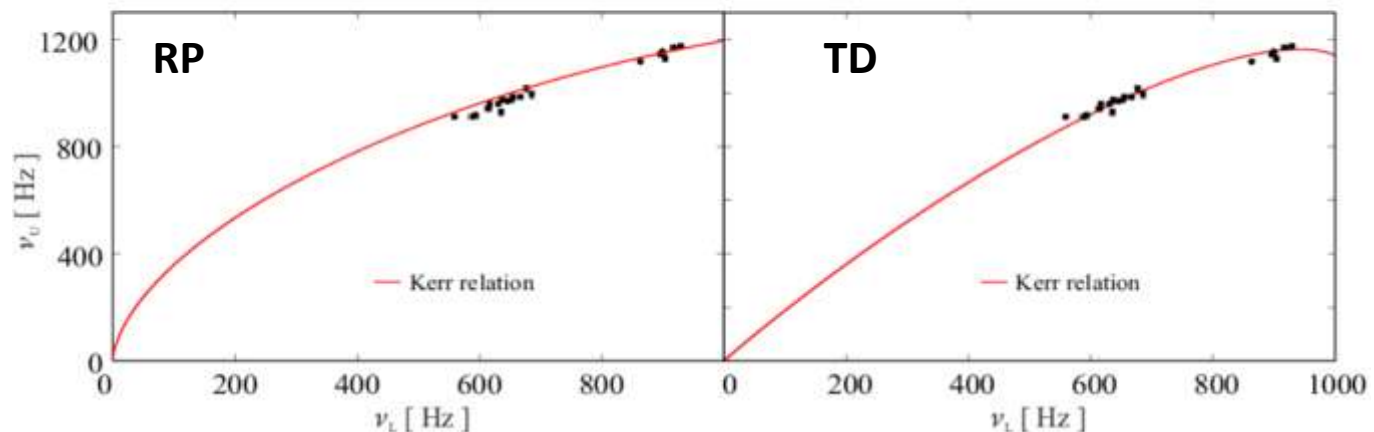
-> preferred mass-spin relations.



Parameters



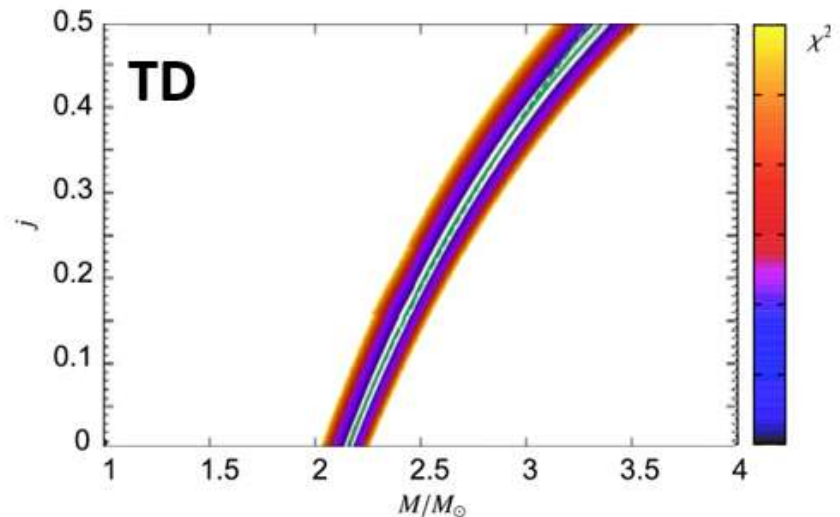
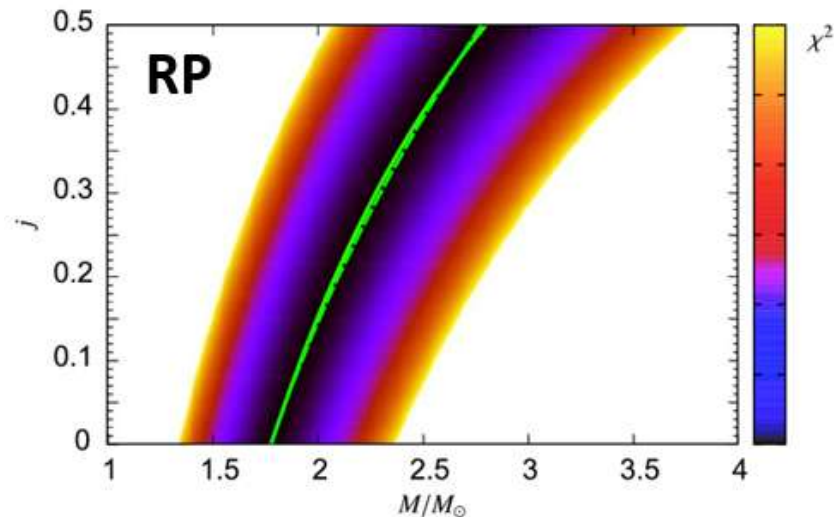
Fits
(4U 1636-53):



3. Orbital models of HF QPOs and NS parameters

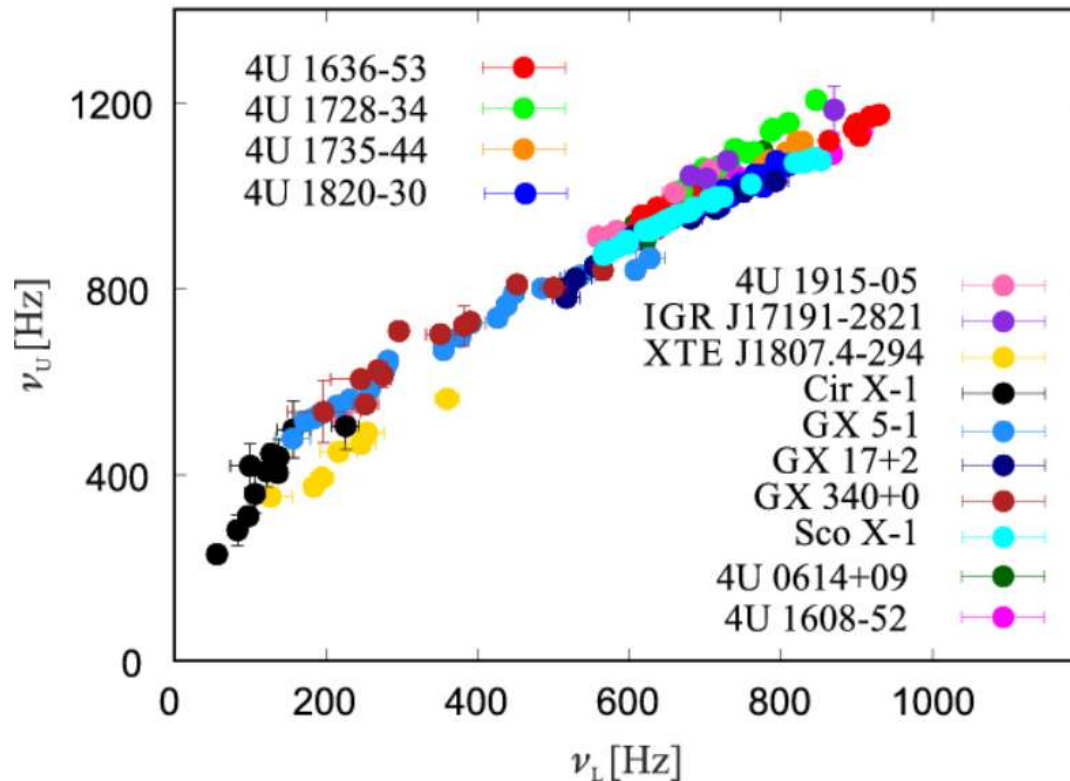
- In the series of works (Torok et al., ApJ, 2010, 2012, 2016) the effective degeneracy between various parameters of several orbital QPO models has been discussed. Within this degeneracy, each combination of NS mass M , angular momentum j and quadrupole moment q corresponds to a certain value of a single generalized parameter \mathcal{M} , e.g., non-rotating NS mass. **This degeneracy seems to be a generic property of (geodesic) QPO models.**

$$\nu_L = \nu_L(\nu_U, \mathcal{M}), \quad \nu_U = \nu_U(\nu_L, \mathcal{M})$$



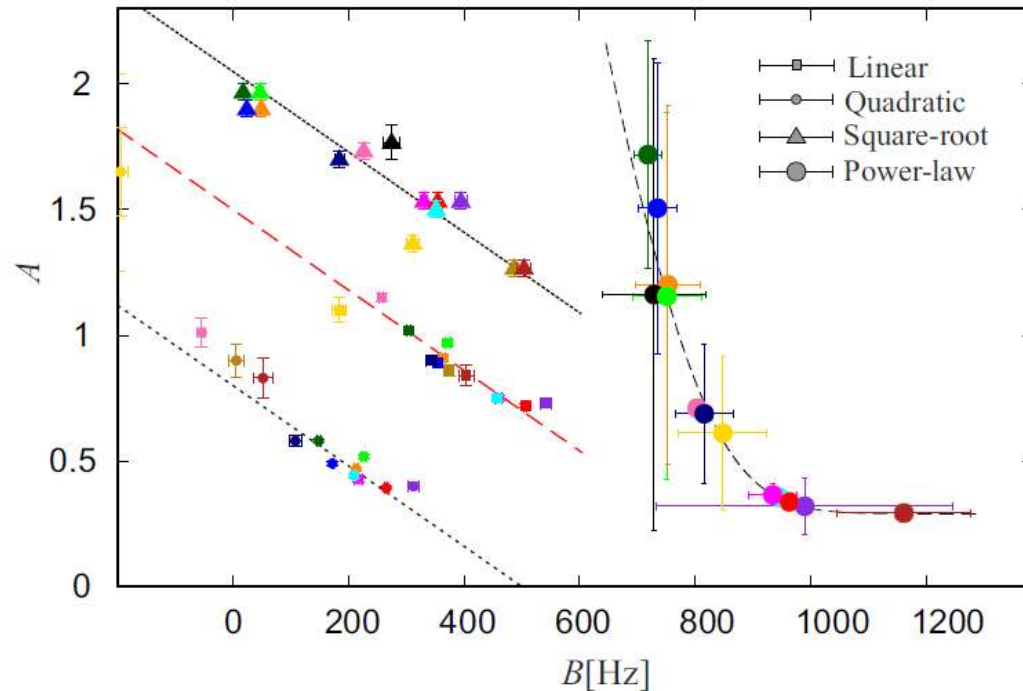
4. Parameters of phenomenological relations

- Parameters of various ad-hoc fitting relations



4. Parameters of phenomenological relations

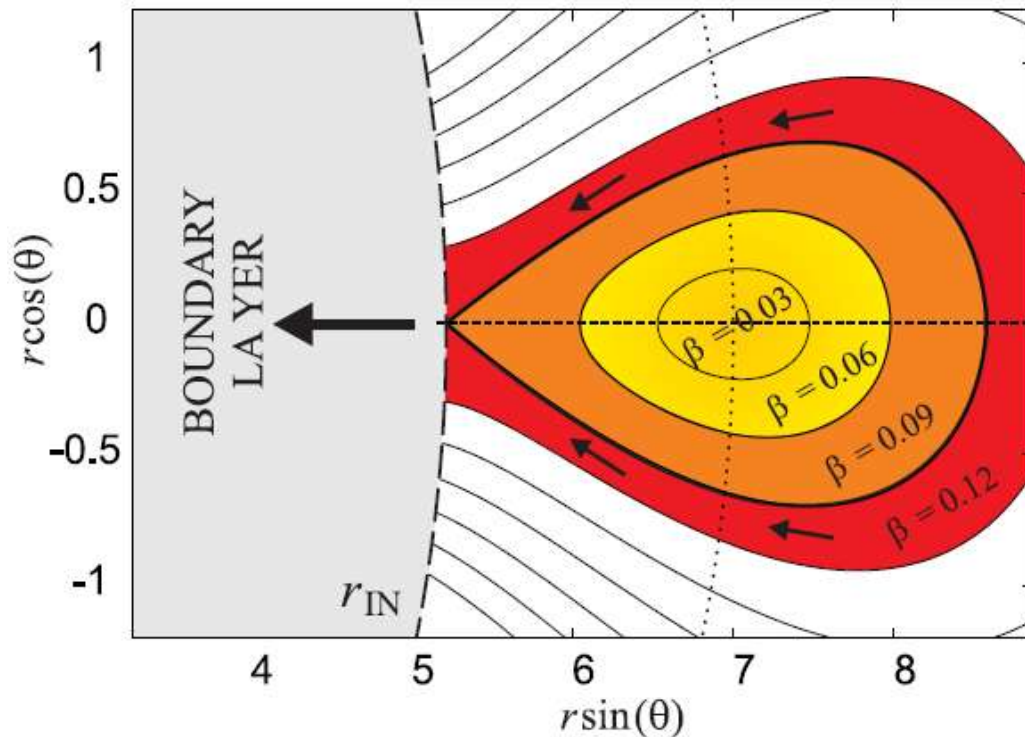
- Parameters of various ad-hoc fitting relations are strongly correlated.



- Overall, one can expect that (the one) relation between the QPO frequencies should be based mostly on a single M parameter.

5. Cusp torus model

- Assumption: innermost region of accretion flow hot enough to form a pressure supported torus of a moderate thickness.
- Modulation and observable frequencies: global Keplerian motion and accretion flow modulation (frequencies of torus oscillations).



Torok et al., 2016, MNRAS Letters

5. Cusp torus model

Mode identification

$$\nu_U \equiv \nu_K(r_0), \quad \nu_L \equiv \nu_{R,-1}(r_0, \beta)$$

- Keplerian motion
- Lowest order non-axisymmetric radial oscillations

Cusp torus configuration

$$\beta(r_0) \doteq \beta_c(r_0)$$

For a fixed spacetime geometry, observable frequencies are functions of torus position only !

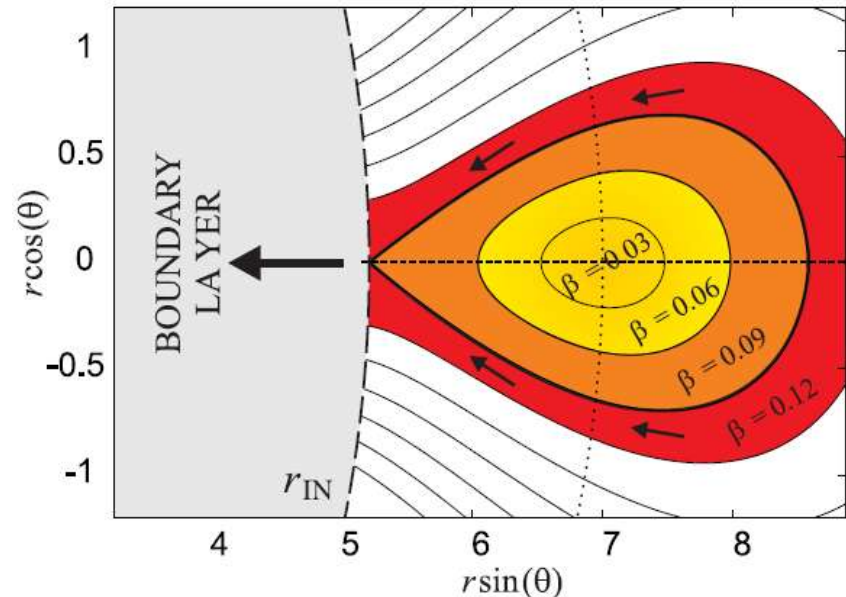
$$\nu_u \equiv \nu_K(r_0), \quad \nu_l \equiv \nu_{R,-1}[r_0, \beta_c(r_0)]$$

$$\nu_r = \left(1 - \frac{6M}{r} + \frac{8jM^{3/2}}{r^{3/2}} - \frac{3j^2M^2}{r^2}\right)^{1/2} \nu_K$$

$$\nu_\theta = \left(1 - \frac{4jM^{3/2}}{r^{3/2}} + \frac{3j^2M^2}{r^2}\right)^{1/2} \nu_K$$

$$\begin{aligned} \nu_{R,m}(r_0, \beta) &= \nu_r(r_0) + m\nu_K(r_0) + \Delta\nu_{R,m}(r_0, \beta), \\ \nu_{V,m}(r_0, \beta) &= \nu_\theta(r_0) + m\nu_K(r_0) + \Delta\nu_{V,m}(r_0, \beta). \end{aligned}$$

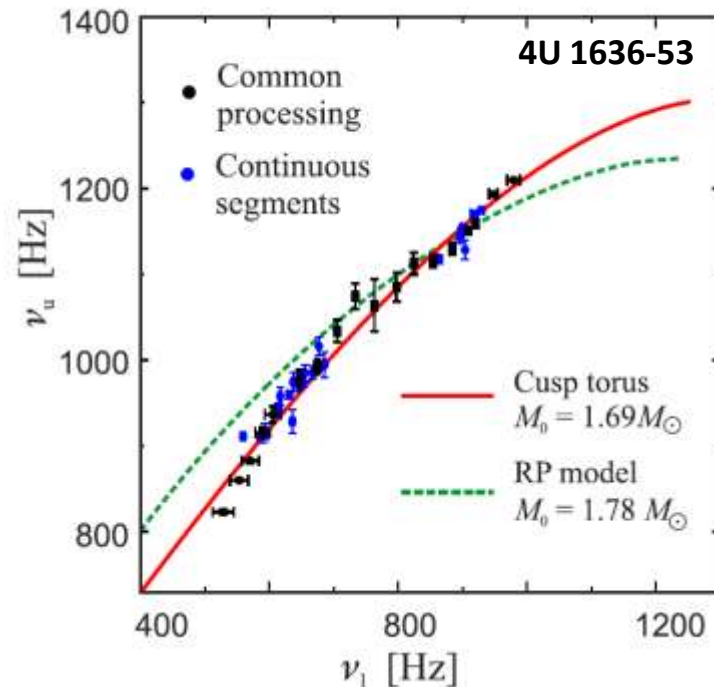
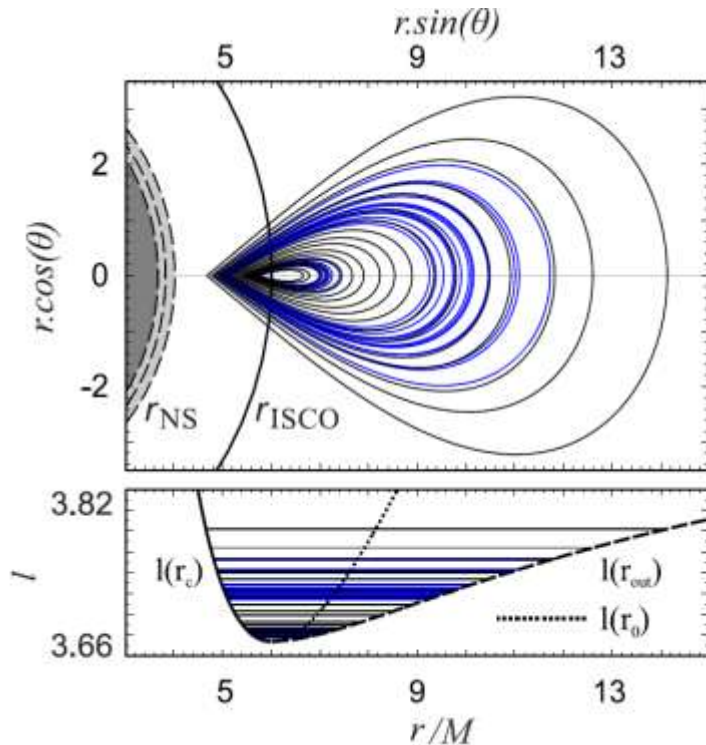
$$\beta_c = \frac{(r_0 - r_c)(r_0 - 2M)^2[r_0r_c - 2M(r_0 + 2r_c)]^{1/2}}{r_cr_0(r_c - 2M)^{1/2}(r_0 - 3M)^{1/2}}$$



5. Cusp torus model

- Neglecting NS rotation, we can fit the data with this model assuming only one free parameter, NS mass M , using the relation

$$\nu_u \equiv \nu_K(r_0), \quad \nu_l \equiv \nu_{R,-1}[r_0, \beta_c(r_0)]$$



Torok et al., 2016, MNRAS Letters

The resulting fit is clearly much better than the fit based on RP model.

5. Cusp torus model

- Approximative formula determining the frequency relation

$$\nu_L = \nu_U \left(1 - \mathcal{B} \sqrt{1 - (\nu_U / \nu_0)^{2/3}} \right)$$

$$\nu_0 \geq \nu_U \geq \nu_L.$$

$$\nu_0 = \nu_{\text{ISCO}} = \frac{1}{6^{3/2}} \frac{c^3}{2\pi G} \frac{1}{M} = 2198 \frac{M_\odot}{M} = 2198 \frac{1}{\mathcal{M}} \text{ [Hz]}$$

$$\nu_L = \nu_U \left(1 - \mathcal{B} \sqrt{1 - 0.0059 (\nu_U \mathcal{M})^{2/3}} \right)$$

5. Cusp torus model

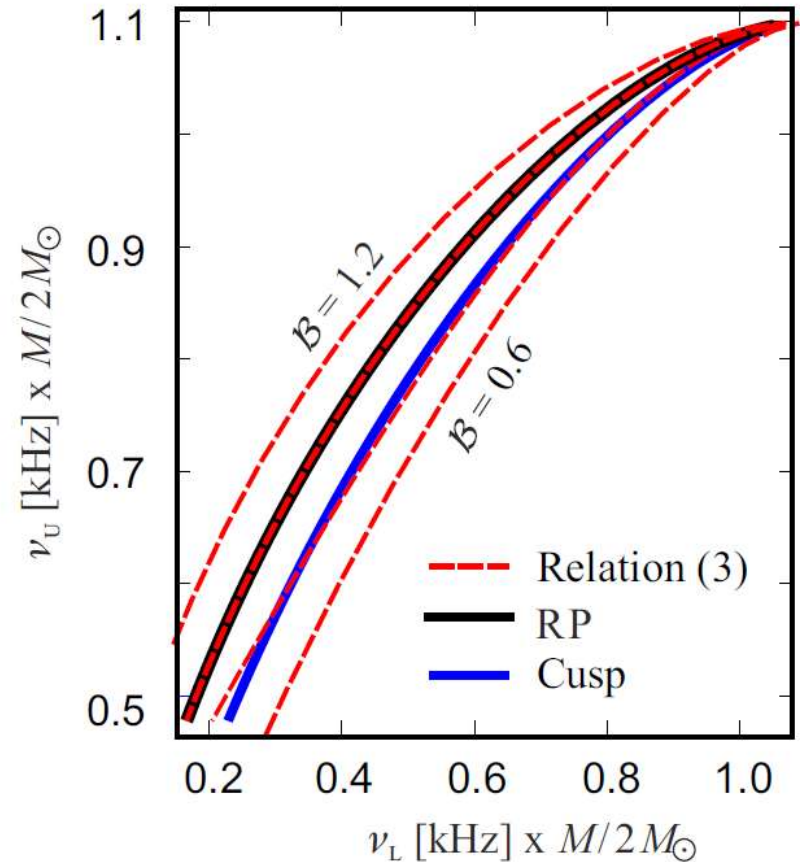
- Approximative formula determining the frequency relation

$$\nu_L = \nu_U \left(1 - B \sqrt{1 - 0.0059 (\nu_U \mathcal{M})^{2/3}} \right)$$

For the choice of

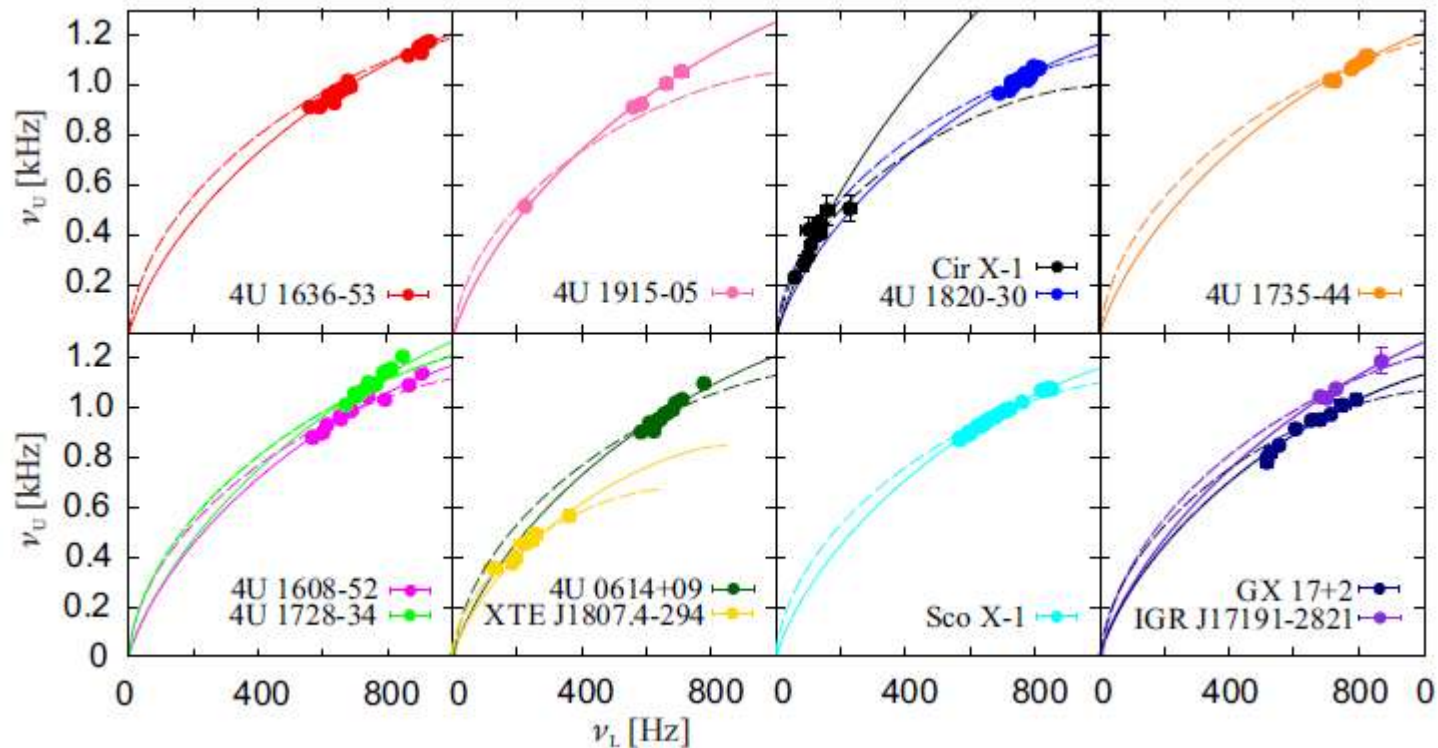
$$B = 0.8,$$

our formula well describes the prediction of CT model, while for $B = 1$ it describes the prediction of RP model.

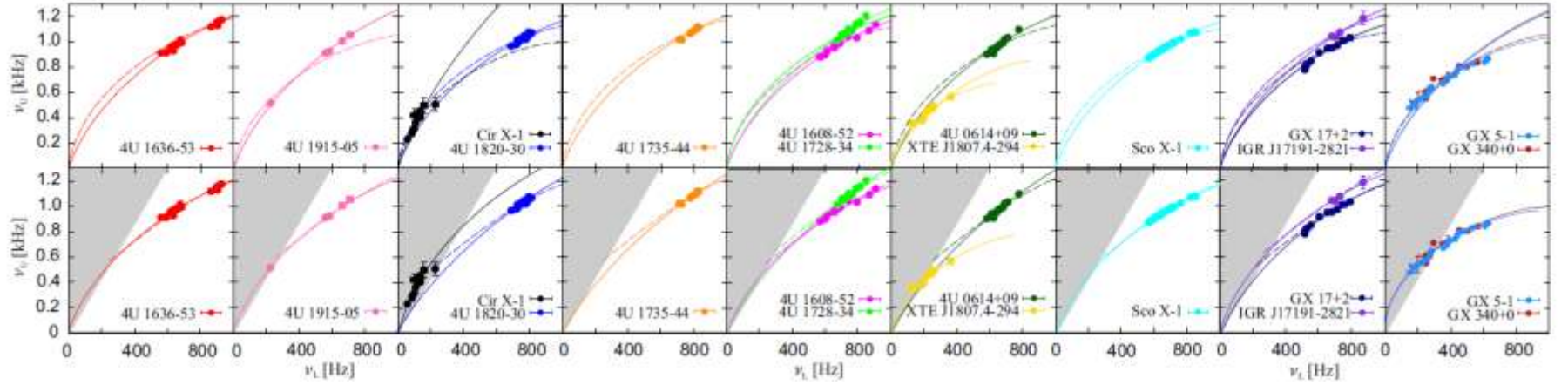


6. Matching the data of individual sources

$$\nu_L = \nu_U \left(1 - \mathcal{B} \sqrt{1 - 0.0059 (\nu_U \mathcal{M})^{2/3}} \right)$$

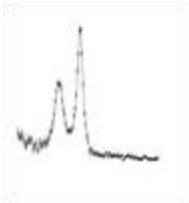


6. Matching the data of individual sources



Source No./ Type ^a	Name	\mathcal{M}	$\frac{\chi^2}{d.o.f.}$	$\mathcal{M}(\mathcal{B})$	\mathcal{B}	$\frac{\chi^2_{\mathcal{M}(\mathcal{B})}}{d.o.f.}$	$\frac{M_{\text{BP}}}{M_{\odot}}$	$\frac{\chi^2_{\text{BP}}}{d.o.f.}$	$\frac{M_{\text{Cusp}}}{M_{\odot}}$	$\frac{\chi^2_{\text{Cusp}}}{d.o.f.}$	Data- points
1/A	4U 1608-52	1.80 ± 0.01	1.6	1.79 ± 0.04	0.79 ± 0.03	1.7	1.94	10.1	1.74 ± 0.01	1.9	12
2/A	4U 1636-53	1.70 ± 0.01	2.0	1.70 ± 0.01	0.8 ± 0.01	2.1	1.79	17.4	1.69 ± 0.01	3.4	22
3/A	4U 1735-44	1.69 ± 0.01	2.1	1.48 ± 0.10	0.61 ± 0.06	1.0	1.81	5.1	1.66 ± 0.01	1.4	8
4/A	4U 1915-05	1.58 ± 0.03	0.8	1.65 ± 0.03	0.82 ± 0.01	0.2	2.09	28.6	— ^b	— ^b	5
5/A	IGR J17191-2821	1.58 ± 0.02	0.6	1.63 ± 0.20	0.85 ± 0.2	0.8	1.76	0.6	1.52 ± 0.02	0.6	4
6/Z	GX 17+2	1.89 ± 0.02	1.2	1.77 ± 0.07	0.72 ± 0.04	0.8	2.08	5.5	1.83 ± 0.02	0.9	10
7/Z	Sco X-1	1.82 ± 0.01	1.0	1.81 ± 0.01	0.8 ± 0.01	1.0	2.0	24.2	1.76 ± 0.01	2.3	39
8/Z	Cir X-1	0.74 ± 0.10	1.2	1.42 ± 0.5	0.89 ± 0.06	1.1	2.23	1.3	— ^b	— ^b	11
9/P	XTE J1807.4-294	2.61 ± 0.11	0.8	2.85 ± 0.25	0.86 ± 0.07	0.8	3.27	1.4	— ^b	— ^b	7
10/A	4U 1728-34	1.57 ± 0.01	3.2	1.35 ± 0.12	0.65 ± 0.06	2.5	1.74	5.7	1.51 ± 0.01	2.8	15
11/A	4U 0614+09	1.71 ± 0.02	5.1	1.39 ± 0.06	0.62 ± 0.02	1.1	1.90	14.7	1.65 ± 0.01	3.4	13
12/A	4U 1820-30	1.81 ± 0.01	9.3	1.53 ± 0.07	0.58 ± 0.03	3.2	1.93	24.2	1.78 ± 0.01	6.4	23
13/Z	GX 340+0	1.62 ± 0.08	4.2	2.23 ± 0.10	1.10 ± 0.08	1.6	2.07	1.8	— ^b	— ^b	12
14/Z	GX 5-1	1.65 ± 0.10	16.7	2.31 ± 0.04	1.11 ± 0.02	1.5	2.13	3.1	— ^b	— ^b	21

6. Conclusions



$$\nu_L = \nu_U \left(1 - B \sqrt{1 - 0.0059 (\nu_U \mathcal{M})^{2/3}} \right)$$

- For 9 sources, our relation well reproduces the data for $B = 0.8$ (the choice associated to the CT model).
- When we consider B as a free parameter, we obtain good fits for each of the 14 considered sources.
- *We note that we have not been able to reproduce the data for any significant group of sources assuming B as a free and M as a fixed parameter.*
- Larger deviations from the case of $B = 0.8$ can have a direct physical interpretation (further non-geodesic effects).
- The CT model consideration agrees with a general interpretation, in which the M parameter represents the main parameter reflecting the spacetime geometry given by the NS mass and spin, while the B parameter reflects the additional stable factors.

7. Acknowledgements

We would like to acknowledge the Czech Science Foundation grant No. 17-16287S, the INTER-TRANSFER project No. LTT17003, the INTER-INFORM project no. LTI17018 and the internal SU grant No. SGS/15/2016.