Pulsar braking: Time dependent moment of inertia?

Martin Urbanec

Institute of Physics and Research Centre for Computational Physics and Data Processing,
Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, CZ-746 01 Opava
e-mail: *martin.urbanec@fpf.slu.cz*

INSTITUTE OF PHYSICS
SU OPAVA

SILESIAN UNIVERSITY IN OPAVA

MINISTRY OF EDUCATION, YOUTH AND SPORTS

Abstract

Pulsars rotate with extremely stable rotational frequency enabling one to measure its first and second time derivatives. These observed values can be combined to the so-called braking index. However observed values of braking index differ from the theoretical value of 3 corresponding to braking by magnetic dipole radiation being the dominant theoretical model. Such a difference can be explained by contribution of other mechanism like pulsar wind or quadrupole radiation, or by time dependency of magnetic field or moment of inertia. In this presentation we focus on influence of time dependent moment of inertia on the braking index. We will also discuss possible physical models for time-dependence of moment of inertia.

Introduction

Pulsars are neutron stars rotating with very stable rotational frequency that is slowly decreasing in time. The stability of spin-down is high enough that spin-down rate can be measured very precisely and in the case of young pulsars the time evolution of spin-down rate (i.e. second time derivative of rotational frequency) can be obtained from observations as well. In the case of older pulsars the second time-derivative of rotational frequency can not be measured precisely since it is hidden under timing noise.

Why time dependent moment of inertia?

There exist several reasons why moment of inertia can be time-dependent.

- The star can be transforming to the strange star
- The star can be rotating differentially at the beginning and undergoing transition to uniformly rotating body
- The core can be more/less coupled to the crust because of superfluid components in the crust-core boundary.

Spin-down of pulsars

The energy radiated by rotating dipole per second can be written as

$$\dot{E} = -\frac{2}{3}B^2R^6\sin^2\alpha\Omega^4, \tag{1}$$

where B is the magnetic field strength at the magnetic pole of the neutron star, R is the radius of the neutron star at the pole, α is the angle between rotational axis and dipole moment, and $\Omega = 2\pi\nu$ is the angular velocity of the star. This is causing the loss of kinetic energy of the rotating neutron star

$$\dot{E} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}I\Omega^2\right] = \frac{1}{2}\Omega^2\dot{I} + I\Omega\dot{\Omega}, \tag{2}$$

where $I = I(t)$ is the moment of inertia of the body and is in general time dependent. Combining previous two equations we find modified braking law

$$\dot{\Omega} = -\frac{C}{I}\Omega^3 - \frac{\Omega\dot{I}}{2I}, \tag{3}$$

where $C = 2/3B^2R^6\sin^2\alpha$ and the second term is modification due to time-dependent moment of inertia. The second time-derivative of angular velocity then reads

$$\ddot{\Omega} = \frac{C}{I^2}\Omega^3\dot{I} - \frac{3C}{I}\Omega^2\dot{\Omega} + \frac{\Omega}{2I^2}\dot{I}^2 - \frac{\Omega}{2I}\ddot{I} - \frac{\dot{I}\dot{\Omega}}{2I}. \tag{4}$$

The braking index n can then be written as

$$n = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2} = 3 - \frac{\Omega^2\ddot{I}}{2\dot{\Omega}^2 I} \tag{5}$$

We can see, that deviation from canonical value of 3 is given by second time derivative of moment of inertia.

Time dependent moment of inertia

We will construct simple analytical model for time-dependent moment of inertia. We assume that

- moment of inertia is time dependent since the creation of pulsar
- after its birth the changes of moment of inertia were very slow, i.e. $\dot{I} = 0$ at the birth of the pulsar
- we focus on leading order in time dependency
- moment of inertia is changing since the birth of the pulsar

All of the above assumptions are satisfied by

$$I(t) = I_0(1 + \beta t^2), \tag{6}$$

where β is some parameter.

We now put the above formula to formula for braking index

$$n = 3 - \frac{\Omega^2 I_0 2\beta}{2\dot{\Omega}^2 I_0(1 + \beta t^2)}, \tag{7}$$

and compare the result with observations. For time t we use characteristic age of the pulsar $t = \tau_c = \Omega/2\dot{\Omega}$ and for n, Ω, $\dot{\Omega}$ we use currently observed values.

Results

In the following table we calculate how big change in moment of inertia of the pulsar is needed to explain currently observed braking indexes. We take the values from recently work of Espinoza, Lyne and Stappers [1]

Pulsar	ν $[\mathrm{s}^{-1}]$	$\dot{\nu}$ $[10^{-11}\mathrm{s}^{-2}]$	n	$\tau_c = \nu/2\dot{\nu}$ $[10^3\mathrm{yr}]$	I/I_0
J0537-6910	62.018	-19.9374	1.2	4.93	1.81
B0531+21	29.946	-37.7535	2.342(1)	1.26	1.20
B0540-69	19.775	-18.7272	2.13(1)	1.67	1.27
J1833-1034	16.159	-5.275	1.857(1)	4.85	1.40
B0833-45	11.200	-1.5660	1.7(2)	11.3	1.48
B1823-13	9.8549	-0.7313	2.2(6)	21.4	1.25
B1800-21	7.4825	-0.7528	1.9(5)	15.8	1.37
B1509-58	6.6115	-6.6944	2.832(3)	1.56	1.04
J1846-0258	3.0621	-6.664	2.65(1)[1]	0.73	1.10
J1119-6127	2.4473	-2.24050	2.684(2)	1.61	1.09
J1208-6238	2.2697	-1.6843	2.598(1)	2.67	1.11
J1734-3333	0.8551	-0.1667	0.9(2)	8.13	2.11

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